today:

quiz 1: algebra and differentiation review
§ 5.3 - the fundamental theorem of calculus
§ 5.4 - indefinite integrals

wednesday:

mslc: riemann sums workshop

thursday:

webwork 0 due
§ 5.5 - substitution
mslc: fundamental theorem of calculus workshop

friday:

mslc: webwork workshop
webwork 1 due

tuesday:

homework 1 due

Use these to evaluate some integrals, say int from 0 to 1 of x (1-x)

Do a geometry-based problem with f that goes negative.

formulae

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} c = n\,c$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} c\,a_i = c\sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2 \qquad \sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

last time...

we used the definite integral to find the area under the curve. For example, the area under

$$f(x) = x^2 + 1$$

from 0 to 1 is

$$\int_0^1 (x^2+1)\mathrm{d}x = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\left(\frac{i}{n}\right)^2 + 1 \right) = \frac{4}{3}$$

but...

what if I changed my mind? Suppose I now want to know the area between 0 and 2. Wouldn't it be great if we didn't have to start over?

area so far

f is a function, so I am free to integrate using any variable, t, x, etc...

if we had an explicit formula for g, then we can find the area from 0 to anywhere.

Functions like g are common: erf (stat, engineering), Si (electrical engineering), Fresnel (optics)

since the area under $f(x)$ from 0 to b is

$$\int_0^b f \;\, t \quad t$$

we can consider the new function $g(x)$ which represents the area from 0 to x:

$$g(x) := \int_0^x f(t)\mathrm{d}t$$

it gets better

suppose instead now I just want to know the area between 1 and 2. Fear not. All is not lost. Since the area from 0 to 2 is the sum of the area from 0 to 1 and the area from 1 to 2,

$$\int_0^2 f(t)\mathrm{d}t = \int_0^1 f(t)\mathrm{d}t + \int_1^2 f(t)\mathrm{d}t$$

and thus

$$\int_1^2 f(t)\mathrm{d}t = \int_0^2 f(t)\mathrm{d}t - \int_0^1 f(t)\mathrm{d}t = g(2) - g(1)$$

fundamental theorem of calculus

Suppose f is continuous on $[a, b]$.

If $g(x) = \int_a^x f(t)\mathrm{d}t$, then $g'(x) = f(x)$.

$\int_a^b f(x)\mathrm{d}x = F(b) - F(a)$, where F is any antiderivative of f, that is $F' = f$.

Proofs are in the book in §5.3. Calculate derivatives of integrals. Use f(x)=x so g is easy to find graphically. Show more complicated expressions. Include one requiring the chain rule.

State the second part as "The Net Change Theorem." i.e. the integral of F' is F(b)-F(a)... that is, the integral of a rate of change is the net change.

Antiderivative == indefinite integral. Use derivative rule for x^n to find antiderivative rule for x^n. Make table of indefinite integrals, practice.

next time

- webwork 0 due thursday, webwork 1 due friday
- read § 5.5
- start paper homework 1
- we will discuss the substitution rule and practice taking integrals